

Subjective Problems

PROBLEM

Find the value of : $\cos(2\cos^{-1}x + \sin^{-1}x)$ at $x = \frac{1}{5}$, where

$$0 \leq \cos^{-1} x \leq \pi \text{ and } -\pi/2 \leq \sin^{-1} x \leq \pi/2.$$

(1981 - 2 Marks)

SOLUTION

We have $\cos(2\cos^{-1}x + \sin^{-1}x)$

$$= \cos(\cos^{-1}x + \cos^{-1}x + \sin^{-1}x)$$

$$= \cos(\cos^{-1}x + \pi/2)$$

{Using $\cos^{-1}x + \sin^{-1}x = \pi/2$ }

$$= -\sin(\cos^{-1}x)$$

$$= -\sqrt{1 - \cos^2(\cos^{-1}x)} = -\sqrt{1 - [\cos(\cos^{-1}x)]^2}$$

$$= -\sqrt{1 - x^2} = -\sqrt{1 - 1/25} \quad [\text{for } x = 1/5]$$

$$= -\frac{\sqrt{24}}{5} = \frac{-2\sqrt{6}}{5}$$