

PROBLEM

The number of real solutions of

$$\tan^{-1} \sqrt{x(x+1)} + \sin^{-1} \sqrt{x^2 + x + 1} = \pi/2 \text{ is}$$

(1999 - 2 Marks)

- (a) zero (b) one (c) two (d) infinite

SOLUTION

$$(c) \quad \tan^{-1} \sqrt{[x(x+1)]} = \pi/2 - \sin^{-1} \sqrt{(x^2 + x + 1)}$$

$$\Rightarrow \tan^{-1} \sqrt{x(x+1)} = \cos^{-1} \sqrt{x^2 + x + 1}$$

$$\Rightarrow \cos^{-1} \frac{1}{\sqrt{x^2 + x + 1}} = \cos^{-1} \sqrt{x^2 + x + 1}$$

$$\Rightarrow x^2 + x + 1 = 1 \Rightarrow x(x+1) = 0$$

$$\Rightarrow x = 0, -1 \text{ are the only real solutions.}$$