MCQs with One Correct Answer

PROBLEM

If $\sin^{-1}\left(x - \frac{x^2}{2} + \frac{x^3}{4} - \ldots\right)$

$$+ \cos^{-1}\left(x^2 - \frac{x^4}{2} + \frac{x^6}{4} - \ldots\right) = \frac{\pi}{2}$$

for $0 < |x| < \sqrt{2}$, then x equals *(2001S)*

(a) 1/2 (b) 1 (c) –1/2 (d) –1

SOLUTION

(b) $\sin^{-1}\left(x-\frac{x^2}{2}+\frac{x^3}{3}\ldots\right)+\cos^{-1}\left(x^2-\frac{x^4}{2}+\frac{x^6}{4}\ldots\right)=\frac{\pi}{2}$

$$\Rightarrow \quad \cos^{-1}\left(x^2-\frac{x^4}{2}+\frac{x^6}{4}\ldots\right)=\frac{\pi}{2}-\sin^{-1}\left(x-\frac{x^2}{2}+\frac{x^3}{4}\ldots\right)$$

$$\Rightarrow \quad \cos^{-1}\left(x^2-\frac{x^4}{2}+\frac{x^6}{4}\ldots\right)=\cos^{-1}\left(x-\frac{x^2}{2}+\frac{x^3}{4}\ldots\right)$$

$$\Rightarrow \quad x^2-\frac{x^4}{2}+\frac{x^6}{4}\ldots=x-\frac{x^2}{2}+\frac{x^3}{4}\ldots$$

On both sides we have G.P. of infinite terms.

$$\therefore \quad x^2 \quad : \frac{x}{1-\left(-\frac{x}{2}\right)} \Rightarrow \frac{2x^2}{2+x^2}=\frac{2x}{2+x}$$

$$\Rightarrow \quad 2x+x^3=2x^2+x^3 \Rightarrow x(x-1)=0$$

$$\Rightarrow \quad x=0,1 \text{ but } 0<|x|<\sqrt{2} \Rightarrow x=1.$$