

JEE ADVANCED/IIT-JEE

Fill in the Blanks

PROBLEM

The greater of the two angles $A = 2 \tan^{-1} (2\sqrt{2}-1)$ and $B = 3 \sin^{-1} (1/3) + \sin^{-1} (3/5)$ is _____.

(1989 - 2 Marks)

SOLUTION

We have

$$A = 2 \tan^{-1}(2\sqrt{2} - 1) = 2 \tan^{-1}(2 \times 1.414 - 1)$$

$$= 2 \tan^{-1}(1.828) > 2 \tan^{-1} \sqrt{3} = 2\pi/3$$

$$\Rightarrow A > 2\pi/3 \quad \dots(1)$$

$$\text{Also } B = 3 \sin^{-1}(1/3) + \sin^{-1}(3/5)$$

$$= \sin^{-1} \left[3 \times \frac{1}{3} - 4 \times \frac{1}{27} \right] + \sin^{-1}(3/5)$$

$$= \sin^{-1} \frac{23}{27} + \sin^{-1}(0.6) = \sin^{-1}(0.852) + \sin^{-1}(0.6)$$

$$< \sin^{-1}(\sqrt{3}/2) + \sin^{-1}(\sqrt{3}/2) = 2\pi/3$$

$$\Rightarrow B < 2\pi/3 \quad \dots(2)$$

From (1) and (2) we conclude $A > B$.