

31. If $\sin^{-1} \frac{2a}{1+a^2} + \cos^{-1} \frac{1-a^2}{1+a^2} = \tan^{-1} \frac{2x}{1-x^2}$, where $a, x \in [0, 1)$ then the value of x is

- (a) 0 (b) $\frac{a}{2}$ (c) a (d) $\frac{2a}{1-a^2}$

Sol. (d) We have, $\sin^{-1} \frac{2a}{1+a^2} + \cos^{-1} \frac{1-a^2}{1+a^2} = \tan^{-1} \frac{2x}{1-x^2}$

$$\Rightarrow 2 \tan^{-1} a + 2 \tan^{-1} a = 2 \tan^{-1} x$$

$$\Rightarrow 2 \tan^{-1} a = \tan^{-1} x$$

$$\Rightarrow \tan^{-1} \frac{2a}{1-a^2} = \tan^{-1} x$$

$$\Rightarrow x = \frac{2a}{1-a^2}$$

$$\left[\begin{array}{l} \because 2 \tan^{-1} x = \tan^{-1} \frac{2x}{1-x^2} \\ 2 \tan^{-1} x = \sin^{-1} \frac{2x}{1+x^2} \\ 2 \tan^{-1} x = \cos^{-1} \frac{1-x^2}{1+x^2} \end{array} \right]$$