

33. The value of the expression $\tan\left(\frac{1}{2}\cos^{-1}\frac{2}{\sqrt{5}}\right)$ is

- (a) $2\sqrt{5}$ (b) $\sqrt{5}-2$ (c) $\frac{\sqrt{5}-2}{2}$ (d) $5-\sqrt{2}$

Sol. (b) We have $\tan\left(\frac{1}{2}\cos^{-1}\frac{2}{\sqrt{5}}\right)$

$$\text{Let } \frac{1}{2}\cos^{-1}\frac{2}{\sqrt{5}} = \theta$$

$$\Rightarrow \cos^{-1}\frac{2}{\sqrt{5}} = 2\theta$$

$$\Rightarrow \cos 2\theta = \frac{2}{\sqrt{5}}$$

$$\therefore 2\cos^2\theta - 1 = \frac{2}{\sqrt{5}}$$

$$\Rightarrow \cos^2\theta = \frac{1}{2} + \frac{1}{\sqrt{5}}$$

$$\Rightarrow \cos\theta = \sqrt{\frac{1}{2} + \frac{1}{\sqrt{5}}}$$

$$\therefore \tan\theta = \frac{\sin\theta}{\cos\theta}$$

$$= \frac{\sqrt{\frac{1}{2} - \frac{1}{\sqrt{5}}}}{\sqrt{\frac{1}{2} + \frac{1}{\sqrt{5}}}} = \sqrt{\frac{\sqrt{5}-2}{\sqrt{5}+2}} = \sqrt{\frac{(\sqrt{5}-2)^2}{(\sqrt{5}+2)(\sqrt{5}-2)}} = \sqrt{5}-2.$$