

Long Answer Type Questions

12. Prove that $\tan^{-1} \frac{\sqrt{1+x^2} + \sqrt{1-x^2}}{\sqrt{1+x^2} - \sqrt{1-x^2}} = \frac{\pi}{4} + \frac{1}{2} \cos^{-1} x^2$.

Sol. We have $\tan^{-1} \left[\frac{\sqrt{1+x^2} + \sqrt{1-x^2}}{\sqrt{1+x^2} - \sqrt{1-x^2}} \right]$

Let $x^2 = \cos 2\theta \Rightarrow \theta = \frac{1}{2} \cos^{-1} x^2$

\therefore L.H.S. = $\tan^{-1} \left[\frac{\sqrt{1+\cos 2\theta} + \sqrt{1-\cos 2\theta}}{\sqrt{1+\cos 2\theta} - \sqrt{1-\cos 2\theta}} \right]$

= $\tan^{-1} \left[\frac{\sqrt{2 \cos^2 \theta} + \sqrt{2 \sin^2 \theta}}{\sqrt{2 \cos^2 \theta} - \sqrt{2 \sin^2 \theta}} \right]$

= $\tan^{-1} \left[\frac{\sqrt{2} \cos \theta + \sqrt{2} \sin \theta}{\sqrt{2} \cos \theta - \sqrt{2} \sin \theta} \right]$

= $\tan^{-1} \left[\frac{1 + \tan \theta}{1 - \tan \theta} \right]$

= $\tan^{-1} \left[\frac{\tan \frac{\pi}{4} + \tan \theta}{1 - \tan \frac{\pi}{4} \tan \theta} \right]$

= $\tan^{-1} \left(\tan \left(\frac{\pi}{4} + \theta \right) \right) = \frac{\pi}{4} + \theta = \frac{\pi}{4} + \frac{1}{2} \cos^{-1} x^2$