

11. Solve the equation $\cos(\tan^{-1}x) = \sin\left(\cot^{-1}\frac{3}{4}\right)$.

Sol. We have, $\cos(\tan^{-1}x) = \sin\left(\cot^{-1}\frac{3}{4}\right)$

L.H.S. = $\cos(\tan^{-1}x)$

$$= \cos\left(\cos^{-1}\frac{1}{\sqrt{x^2+1}}\right)$$

$$= \frac{1}{\sqrt{x^2+1}}$$

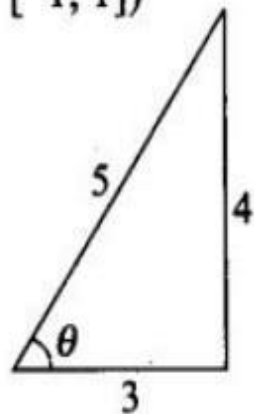
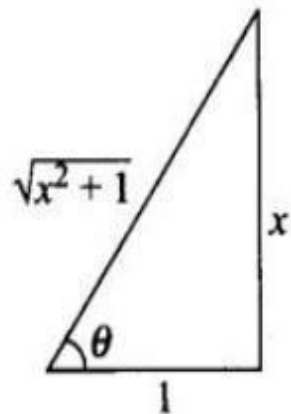
($\because \cos(\cos^{-1}x) = x, x \in [-1, 1]$)

R.H.S. = $\sin\left(\cot^{-1}\frac{3}{4}\right)$

$$= \sin\left(\sin^{-1}\frac{4}{5}\right)$$

$$= \frac{4}{5}$$

($\because \sin(\sin^{-1}x) = x, x \in [-1, 1]$)



\therefore From given equation we get $\frac{1}{\sqrt{x^2+1}} = \frac{4}{5}$

$$\Rightarrow 16(x^2 + 1) = 25$$

$$\Rightarrow 16x^2 = 9$$

$$\Rightarrow x^2 = \frac{9}{16}$$

$$\therefore x = \pm \frac{3}{4}$$