Example 37 The value of $\tan\left(\cos^{-1}\frac{3}{5}+\tan^{-1}\frac{1}{4}\right)$ is

(A) $\frac{19}{8}$ (B) $\frac{8}{19}$ (C) $\frac{19}{12}$ (D) $\frac{3}{4}$

Solution (A) is the correct answer. $\tan\left(\cos^{-1}\frac{3}{5}+\tan^{-1}\frac{1}{4}\right)=\tan\left(\tan^{-1}\frac{4}{3}+\tan^{-1}\frac{1}{4}\right)$

8. **Find the value of the expression $\sin\left(2\tan^{-1}\frac{1}{3}\right)+\cos(\tan^{-1}2\sqrt{2})$.**

Sol. We have, $\sin\left(2\tan^{-1}\frac{1}{3}\right)+\cos(\tan^{-1}2\sqrt{2})$

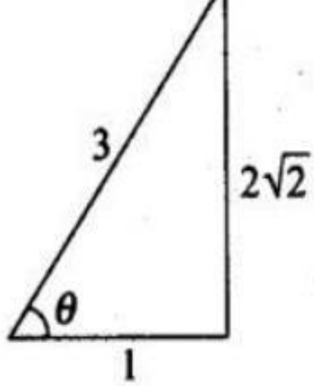
$$\sin\left(2\tan^{-1}\frac{1}{3}\right)=\sin\left(\sin^{-1}\frac{2\times\frac{1}{3}}{1+\left(\frac{1}{3}\right)^2}\right)\quad\left(\because 2\tan^{-1}x=\sin^{-1}\frac{2x}{1+x^2}\right)$$

$$=\sin\left(\sin^{-1}\frac{2/3}{10/9}\right)$$

$$=\sin\left(\sin^{-1}\frac{3}{5}\right)=\frac{3}{5}\quad(\because \sin(\sin^{-1}x)=x, x\in[-1,1])$$

$$\cos(\tan^{-1}2\sqrt{2})=\cos\left(\cos^{-1}\frac{1}{3}\right)=\frac{1}{3}$$

$$(\because \cos(\cos^{-1}x)=x, x\in[-1,1])$$



$$\therefore\quad \sin\left(2\tan^{-1}\frac{1}{3}\right)+\cos(\tan^{-1}2\sqrt{2})$$

$$=\frac{3}{5}+\frac{1}{3}=\frac{9+5}{15}=\frac{14}{15}$$