Long Answer (L.A.)

Example 13 Prove that $2\sin^{-1}\frac{3}{5} - \tan^{-1}\frac{17}{31} = \frac{\pi}{4}$

Solution Let $\sin^{-1}\frac{3}{5} = \theta$, then $\sin\theta = \frac{3}{5}$, where $\theta \in \left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$

Thus $\tan\theta = \frac{3}{4}$, which gives $\theta = \tan^{-1}\frac{3}{4}$.

Therefore, $2\sin^{-1}\frac{3}{5} - \tan^{-1}\frac{17}{31}$

$$= 2\theta - \tan^{-1}\frac{17}{31} \quad = 2\tan^{-1}\frac{3}{4} - \tan^{-1}\frac{17}{31}$$

$$= \tan^{-1}\left(\frac{2.\frac{3}{4}}{1-\frac{9}{16}}\right) - \tan^{-1}\frac{17}{31} = \tan^{-1}\frac{24}{7} - \tan^{-1}\frac{17}{31}$$

$$= \tan^{-1}\left(\frac{\frac{24}{7}-\frac{17}{31}}{1+\frac{24}{7}.\frac{17}{31}}\right) = \frac{\pi}{4}$$

$$= \tan\tan^{-1}\left(\frac{\frac{4}{3}+\frac{1}{4}}{1-\frac{4}{3}\times\frac{1}{4}}\right) = \tan\tan^{-1}\left(\frac{19}{8}\right) = \frac{19}{8}\cdot$$