

Example 10 Find value of $\tan(\cos^{-1}x)$ and hence evaluate $\tan \cos^{-1} \frac{8}{17}$.

Solution Let $\cos^{-1}x = \theta$, then $\cos \theta = x$, where $\theta \in [0, \pi]$

Therefore, $\tan(\cos^{-1}x) = \tan \theta = \frac{\sqrt{1-\cos^2 \theta}}{\cos \theta} = \frac{\sqrt{1-x^2}}{x}$.

Hence $\tan\left(\cos^{-1}\frac{8}{17}\right) = \frac{\sqrt{1-\left(\frac{8}{17}\right)^2}}{\frac{8}{17}} = \frac{15}{8}$.