Q.5) Solve $\tan^{-1}x + \cot^{-1}(-|x|) = 2\tan^{-1}6x$.

Soln -

Case I : $x < 0$

So, $\tan^{-1}x + \cot^{-1}x = 2\tan^{-1}6x$

$$\frac{\pi}{2} = 2\tan^{-1}6x$$

$$\Rightarrow \tan^{-1}6x = \frac{\pi}{4}$$

$$\Rightarrow 6x = 1$$

$$\Rightarrow x = \frac{1}{6}$$ This is not possible because $x < 0$.

Case II : $x \geq 0$

So, $\tan^{-1}x + \cot^{-1}(-x) = 2\tan^{-1}(6x)$

$$\Rightarrow \tan^{-1}x + \frac{\pi}{2} - \cot^{-1}(x) = 2\tan^{-1}(6x)$$

$$\Rightarrow \frac{\pi}{2} + 2\tan^{-1}x = 2\tan^{-1}(6x)$$

$$\Rightarrow \tan^{-1}6x - \tan^{-1}x = \frac{\pi}{4}$$

$$\Rightarrow \tan^{-1}\left(\frac{6x - x}{1 + (6x)(x)}\right) = \frac{\pi}{4}$$

$$\Rightarrow \frac{5x}{1+6x^2} = 1$$

$$\Rightarrow 6x^2 - 5x + 1 = 0$$

$$\Rightarrow \boxed{x = \frac{1}{2} \text{ OR } \frac{1}{3}}$$