Q.4) Find the sum $\sum_{r=1}^{\infty} \tan^{-1}\left(\frac{2(2r-1)}{4+r^2(r^2-2r+1)}\right)$

$$S_n = \sum_{r=1}^{n} \tan^{-1} \frac{2[r^2-(r-1)^2]}{4+r^2(r-1)^2}$$

$$= \sum_{r=1}^{n} \tan^{-1}\left[\frac{\frac{r^2}{2}-\frac{(r-1)^2}{2}}{1+\frac{r^2}{2}\cdot\frac{(r-1)^2}{2}}\right]$$

$$= \sum_{r=1}^{n} \tan^{-1}\left(\frac{r^2}{2}\right) - \tan^{-1}\left(\frac{(r-1)^2}{2}\right)$$

$$= \tan^{-1}\left(\frac{n^2}{2}\right) - \tan^{-1}\left(\frac{(1-1)^2}{2}\right)$$

$$= \tan^{-1}\left(\frac{n^2}{2}\right)$$

$$S_\infty = \lim_{n\to\infty} S_n = \lim_{n\to\infty} \tan^{-1}\left(\frac{n^2}{2}\right)$$

$$= \frac{\pi}{2}$$

So, $S_\infty = \frac{\pi}{2}$