Q.2) If $a_1, a_2, a_3, \ldots, a_n$ are in A.P, with common difference d, then prove that

$$\tan\left[\tan^{-1}\left(\frac{d}{1+a_1a_2}\right)+\tan^{-1}\left(\frac{d}{1+a_2a_3}\right)+\ldots\tan^{-1}\left(\frac{d}{1+a_{n-1}a_n}\right)\right]$$

$$=\frac{(n-1)d}{1+a_1a_n}$$

Soln-

$$\tan^{-1}\left(\frac{d}{1+a_1a_2}\right)+\tan^{-1}\left(\frac{d}{1+a_2a_3}\right)+\ldots+\tan^{-1}\left(\frac{d}{1+a_{n-1}a_n}\right)$$

$$=\tan^{-1}\left(\frac{a_2-a_1}{1+a_1a_2}\right)+\tan^{-1}\left(\frac{a_3-a_2}{1+a_2a_3}\right)+\ldots+\tan^{-1}\left(\frac{a_n-a_{n-1}}{1+a_{n-1}a_n}\right)$$

$$=\left(\tan^{-1}a_2-\tan^{-1}a_1\right)+\left(\tan^{-1}a_3-\tan^{-1}a_2\right)+\ldots+\left(\tan^{-1}a_n-\tan^{-1}_{n-1}\right)$$

$$=\tan^{-1}a_n-\tan^{-1}a_1$$

$$=\tan^{-1}\left(\frac{a_n-a_1}{1+a_1a_n}\right)=\tan^{-1}\left(\frac{(n-1)d}{1+a_1a_n}\right)$$

Hence,

$$\tan\left[\tan^{-1}\left(\frac{d}{1+a_1a_2}\right)+\tan^{-1}\left(\frac{d}{1+a_2a_3}\right)+\ldots+\tan^{-1}\left(\frac{d}{1+a_{n-1}a_n}\right)\right]=\frac{(n-1)d}{1+a_1a_n}$$