

Solve the following equation for x .

$$\tan^{-1} 2x + \tan^{-1} 3x = \frac{\pi}{4} \quad (1978, 3M)$$

Considering only the principal values of inverse functions, the set $A = \left\{ x \geq 0 : \tan^{-1}(2x) + \tan^{-1}(3x) = \frac{\pi}{4} \right\}$

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- (a) is an empty set
- (b) is a singleton
- (c) contains more than two elements
- (d) contains two elements

Sol- $\tan^{-1} 2x + \tan^{-1} 3x = \frac{\pi}{4}$

$$\Rightarrow \tan \left[\tan^{-1} 2x + \tan^{-1} 3x \right] = \tan \left(\frac{\pi}{4} \right)$$

$$\Rightarrow \frac{2x + 3x}{1 - (2x)(3x)} = 1$$

$$\Rightarrow 5x = 1 - 6x^2$$

$$\Rightarrow 6x^2 + 5x - 1 = 0$$

$$\Rightarrow \text{either } x = \frac{1}{6} \text{ OR } x = -1.$$

But $x = -1$ does not satisfy the eqn.

$$\text{So, } x = \frac{1}{6}.$$