The value of $\cot\left(\sum_{n=1}^{23}\cot^{-1}\left(1+\sum_{k=1}^{n}2k\right)\right)$ is

(JEE Adv. 2013)

(a) $\frac{23}{25}$ (b) $\frac{25}{23}$ (c) $\frac{23}{24}$ (d) $\frac{24}{23}$

Soln-

$$\cot\left[\sum_{n=1}^{23}\cot^{-1}\left(1+\sum_{k=1}^{n}(2k)\right)\right]$$

$$=\cot\left[\sum_{n=1}^{23}\cot^{-1}\left(1+n(n+1)\right)\right]$$

$$=\cot\left[\sum_{n=1}^{23}\tan^{-1}\left(\frac{(n+1)-n}{1+n(n+1)}\right)\right]$$

$$=\cot\left[\sum_{n=1}^{23}\tan^{-1}(n+1)-\tan^{-1}(n)\right]$$

$$=\cot\left[\left(\tan^{-1}2-\tan^{-1}1\right)+\left(\tan^{-1}(3)-\tan^{-1}(2)\right)+\cdots+\left(\tan^{-1}(24)-\tan^{-1}(23)\right)\right]$$

$$=\cot\left(\tan^{-1}24-\tan^{-1}1\right)$$

$$=\cot\left(\tan^{-1}\left[\frac{24-1}{1+(24)(1)}\right]\right)$$

$$=\cot\left[\tan^{-1}\left(\frac{23}{25}\right)\right]$$

$$=\cot\left[\cot^{-1}\left(\frac{25}{23}\right)\right]=\frac{25}{23}$$ Ans