Let a, b, c be positive real numbers. Let

$$\theta = \tan^{-1}\sqrt{\frac{a(a+b+c)}{bc}} + \tan^{-1}\sqrt{\frac{b(a+b+c)}{ca}}$$

$$+ \tan^{-1}\sqrt{\frac{c(a+b+c)}{ab}}.$$

Then $\tan\theta = $ _____

(1981 – 2 Marks)

Sol.- we know that,

$$\because \tan^{-1}x + \tan^{-1}y + \tan^{-1}z = \tan^{-1}\left(\frac{x+y+z-xyz}{1-xy-yz-zx}\right)$$

So, $\tan^{-1}\left(\sqrt{\frac{a(a+b+c)}{bc}}\right) + \tan^{-1}\left(\sqrt{\frac{b(a+b+c)}{ac}}\right) + \tan^{-1}\left(\sqrt{\frac{c(a+b+c)}{ab}}\right)$

$$= \tan^{-1}\left[\frac{\sqrt{a+b+c}\left(\sqrt{\frac{a}{bc}} + \sqrt{\frac{b}{ac}} + \sqrt{\frac{c}{ab}}\right) - (a+b+c)\sqrt{\frac{a+b+c}{abc}}}{1 - \frac{a+b+c}{c} - \frac{a+b+c}{a} - \frac{a+b+c}{b}}\right]$$

$$= \tan^{-1}\left[\frac{(a+b+c)\sqrt{\frac{a+b+c}{abc}} - (a+b+c)\sqrt{\frac{a+b+c}{abc}}}{1 - (a+b+c)\left(\frac{1}{c} + \frac{1}{a} + \frac{1}{b}\right)}\right]$$

$$= \tan^{-1}(0)$$

$$= 0 \qquad \underline{Ans}$$