Example 39 The equation $\tan^{-1}x - \cot^{-1}x = \tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$ has

(A) no solution
(B) unique solution
(C) infinite number of solutions
(D) two solutions

Solution (B) is the correct answer. We have

$$\tan^{-1}x - \cot^{-1}x = \frac{\pi}{6} \text{ and } \tan^{-1}x + \cot^{-1}x = \frac{\pi}{2}$$

Adding them, we get $2\tan^{-1}x = \frac{2\pi}{3}$

$$\Rightarrow \tan^{-1}x = \frac{\pi}{3} \text{ i.e., } x = \sqrt{3}.$$