

Example 19 Solve the equation $\sin^{-1} 6x + \sin^{-1} 6\sqrt{3}x = -\frac{\pi}{2}$

Solution From the given equation, we have $\sin^{-1} 6x = -\frac{\pi}{2} - \sin^{-1} 6\sqrt{3}x$

$$\Rightarrow \sin(\sin^{-1} 6x) = \sin\left(-\frac{\pi}{2} - \sin^{-1} 6\sqrt{3}x\right)$$

$$\Rightarrow 6x = -\cos(\sin^{-1} 6\sqrt{3}x)$$

$$\Rightarrow 6x = -\sqrt{1-108x^2}. \text{ Squaring, we get}$$
$$36x^2 = 1 - 108x^2$$

$$\Rightarrow 144x^2 = 1 \qquad \Rightarrow x = \pm \frac{1}{12}$$