Example 14 Prove that

$$\cot^{-1}7 + \cot^{-1}8 + \cot^{-1}18 = \cot^{-1}3$$

Solution We have

$$\cot^{-1}7 + \cot^{-1}8 + \cot^{-1}18$$

$$= \tan^{-1}\frac{1}{7} + \tan^{-1}\frac{1}{8} + \tan^{-1}\frac{1}{18} \qquad \left(\text{since } \cot^{-1}x = \tan^{-1}\frac{1}{x},\ \text{if } x > 0\right)$$

$$= \tan^{-1}\left(\frac{\frac{1}{7}+\frac{1}{8}}{1-\frac{1}{7}\times\frac{1}{8}}\right) + \tan^{-1}\frac{1}{18} \qquad \left(\text{since } x \,.\, y = \frac{1}{7}\cdot\frac{1}{8} < 1\right)$$

$$= \tan^{-1}\frac{3}{11}+\tan^{-1}\frac{1}{18} = \tan^{-1}\left(\frac{\frac{3}{11}+\frac{1}{18}}{1-\frac{3}{11}\times\frac{1}{18}}\right) \qquad \text{(since } xy < 1\text{)}$$

$$= \tan^{-1}\frac{65}{195} \qquad = \tan^{-1}\frac{1}{3} \quad = \cot^{-1} 3$$