NOTES

f : Trigonometric fns
(sin, cos, tan, cot, sec, cosec)

$f: A \longrightarrow B$

$f^{-1}: B \rightarrow C \subset A$

If $f(\theta) = x$, $\theta \in C$ then, $\theta = f^{-1}(x)$

If $f(\theta) = x$, $\theta \notin C$ then, $\theta = ?$

Example :

$\sin : R \longrightarrow [-1, 1]$
$\sin^{-1} : [-1, 1] \longrightarrow [-\pi/2, \pi/2]$

$\sin\theta = x$, $x \in [-1, 1]$
Find θ, if $\theta \in \left[m\pi - \frac{\pi}{2}, m\pi + \frac{\pi}{2}\right]$ for integer m.

$\sin\theta = x$
— , $\theta \in \left[m\pi - \frac{\pi}{2}, m\pi + \frac{\pi}{2}\right]$

then, $\theta - m\pi \in \left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$ Range set of $\sin^{-1}$

$$\sin(\theta - m\pi) = \sin\theta \cos m\pi + \sin m\pi \cos\theta$$
$$= \sin\theta\, (-1)^m$$

m even : $\sin(\theta - m\pi) = x \Rightarrow \theta = m\pi + \sin^{-1}x$
m odd : $\sin(\theta - m\pi) = -x \Rightarrow \theta = m\pi - \sin^{-1}x$

$\tan(\tan^{-1}x + \tan^{-1}y)$

$$= \frac{\tan(\tan^{-1}x) + \tan(\tan^{-1}y)}{1 - \tan(\tan^{-1}x) . \tan(\tan^{-1}y)}$$

$$= \frac{x+y}{1-xy}$$

Now, if

$xy < 1$, $\tan^{-1}x + \tan^{-1}y = \tan^{-1}\left(\frac{x+y}{1-xy}\right)$

$x>0, y>0, xy>1$, $\tan^{-1}x + \tan^{-1}y = \pi + \tan^{-1}\left(\frac{x+y}{1-xy}\right)$

$x<0, y<0, xy>1$, $\tan^{-1}x + \tan^{-1}y = -\pi + \tan^{-1}\left(\frac{x+y}{1-xy}\right)$

How do we compute -
$\sin^{-1}x + \cos^{-1}y$, using the $\tan^{-1}x + \tan^{-1}y$ type formula.

Can we write a general formula
$\sin^{-1}x + \sin^{-1}y = \sin^{-1}(?) + (?)$

16. a) $\sin^{-1} x = \tan^{-1}(?)$, $|x| \le 1$

let, $\theta = \sin^{-1} x$ $\quad \theta \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

$$\tan\theta = \frac{\sin\theta}{\sqrt{1-\sin^2\theta}} \qquad \because \cos\theta > 0$$

$$\tan\theta = \frac{x}{\sqrt{1-x^2}}$$

$$\tan(\sin^{-1} x) = \frac{x}{\sqrt{1-x^2}}$$

$$\boxed{\sin^{-1} x = \tan^{-1}\left(\frac{x}{\sqrt{1-x^2}}\right)}^{**}$$

Find $\sin^{-1}\left(\frac{1}{3}\right) + \sin^{-1}\left[\frac{2}{3}\left(1 - \frac{1}{\sqrt{8}}\right)\right] = ?$

$$\sin^{-1}\left(\frac{1}{3}\right) = \tan^{-1}\left(\frac{(1/3)}{\sqrt{1-1/9}}\right) = \tan^{-1}\left(\frac{1}{\sqrt{8}}\right)$$

$$\sin^{-1}\left[\frac{2}{3}\left(1-\frac{1}{\sqrt{8}}\right)\right] = \tan^{-1}\left[\frac{\frac{2}{3}\left(1-\frac{1}{\sqrt{8}}\right)}{\sqrt{1 - 2/3\left(1-\frac{1}{\sqrt{8}}\right)^2}}\right] = \tan^{-1}\left(\frac{\sqrt{8}-1}{\sqrt{9+2\sqrt{8}}}\right)$$

$$\sin^{-1}\left(\frac{1}{3}\right) + \sin^{-1}\left[\frac{2}{3}\left(1-\frac{1}{\sqrt{8}}\right)\right] = \tan^{-1}\left(\frac{1}{\sqrt{8}}\right) + \tan^{-1}\left(\frac{\sqrt{8}-1}{\sqrt{9+2\sqrt{8}}}\right)$$

Using $\tan^{-1} x + \tan^{-1} y = \tan^{-1}\left(\frac{x+y}{1-xy}\right)$

$$= \tan^{-1}(1) = \frac{\pi}{4}$$