Example 2: Find the solution of the homogeneous differential equation xCos(y/x).dy/dx = yCos(y/x) + x.

Solution:

The given differential equation is xCos(y/x).dy/dx = yCos(y/x) + x

$$dy/dx = \frac{y\text{Cos}(y/x) + x}{x\text{Cos}(y/x)}$$

$$dy/dx = \frac{x((y/x).\text{Cos}(y/x) + 1)}{x\text{Cos}(y/x)}$$

$$dy/dx = \frac{((y/x).\text{Cos}(y/x) + 1)}{\text{Cos}(y/x)}$$

Here let us substitute y/x =v in the above expression.

$$dy/dx = \frac{v\text{Cos}v + 1}{\text{Cos}v}$$

Here we write y/x = v as y = vx. Differentiating y = vx on both sides we obtain dy/dx = v + x.dv/dx, which is substituted in the above expression.

$$v + x.dv/dx = \frac{vCosv + 1}{Cosv}$$

$$x.dv/dx = \frac{vCosv + 1}{Cosv} - v$$

Here we separate the variables on either side of the equal to symbol.

$x.dv/dx = 1/Cosv$

$Cosv.dv = dx/x$

Integrating this expression on both sides, we have the below expression.

$$\int Cosv.\,dv = \int \frac{1}{x}.\,dx$$

$Sinv = Logx + C$

Here we substitute back $y/x = v$.

$Sin\ y/x = Logx + C$

Therefore, the solution of the homogeneous differential equation is $\sin y/x = \log x + C$.