Example 1: Show that the differential equation $(x - y).dy/dx = (x + 2y)$ is a homogeneous differential equation.

Solution:

$(x - y).dy/dx = (x + 2y)$ is the given differential equation.

To prove that the above differential equation is a homogeneous differential equation, let us substitute $x = \lambda x$, and $y = \lambda y$.

Here we have $F(x, y) = \dfrac{(x + 2y)}{(x - y)}$

$F(\lambda x, \lambda y) = \dfrac{(\lambda x + 2\lambda y)}{(\lambda x - \lambda y)}$

$F(\lambda x, \lambda y) = \dfrac{\lambda(x + 2y)}{\lambda(x - y)} = \lambda^0 f(x, y)$

Therefore, the given differential equation is a homogeneous differential equation.