Example 1: Solve dy/dx = $y^2 - x^2/2xy$?

Solution:

Clearly, since each of the functions ($y^2 - x^2$) and 2xy is a homogeneous function of degree 2, the given equation is homogeneous.

Putting y = vx and dy/dx = v + x dy/dx, the given equation becomes

$v + x \, dv/dx = (v^2x^2 - x^2)/2vx^2$

=> $v + x \, dv/dx = v^2 - 1/2v$ [after dividing ($v^2x^2/2vx^2 - x^2/2vx^2$)]

=> $x \, dv/dx = ((v^2 - 1/2v) - v)$

=> $x \, dv/dx = -(1 + v^2)/2v$

$\Rightarrow 2v/(1+v^2)dv = -1/x\,dx$

$\Rightarrow \int 2v/(1+v^2)dv = -\int 1/x\,dx$ [Integrating both the sides]

$\Rightarrow \log|1+v^2| = -\log|x| + \log C$

$\Rightarrow \log|1+v^2| + \log|x| = \log C$

$\Rightarrow \log|x(1+v^2)| = \log C$

$\Rightarrow x(1+v^2) = \pm C$

$\Rightarrow x(1+v^2) = C_1$

$\Rightarrow x(1+y^2/x^2) = C_1$ [Putting the original value of $v = y/x$]

$\Rightarrow (x^2+y^2) = xC_1$, which is the required solution

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