

The marginal revenue 'y' of output 'q' is given by the equation  $\frac{dy}{dq} = \frac{q^2 + 3y^2}{2qy}$ . Find the total Revenue function when output is 1 unit and Revenue is ₹5.

**Solution:**

Given that  $MR = \frac{dy}{dq} = \frac{q^2 + 3y^2}{2qy}$  (1)

Put  $y = vq$  and  $\frac{dy}{dq} = v + q \frac{dv}{dq}$  in (1)

Now (1) becomes

$$\begin{aligned}v + q \frac{dv}{dq} &= \frac{q^2 + 3v^2 q^2}{2q vq} \\ &= \frac{1 + 3v^2}{2v} \\ q \frac{dv}{dq} &= \frac{1 + 3v^2}{2v} - v \\ &= \frac{1 + 3v^2 - 2v^2}{2v}\end{aligned}$$

$$= \frac{1+v^2}{2v}$$

$$\frac{2v}{1+v^2} dv = \frac{dq}{q}$$

On Integration

$$\int \frac{2v}{1+v^2} dv = \int \frac{dq}{q}$$

$$\log(1+v^2) = \log q + \log c$$

$$1+v^2 = cq$$

Replace

$$v = \frac{y}{q}$$

$$1 + \frac{y^2}{q^2} = cq$$

$$q^2 + y^2 = cq^3 \quad (2)$$

Given output is 1 unit and revenue is ₹5

$$\therefore (2) \Rightarrow 1 + 25 = c \Rightarrow c = 26$$

$\therefore$  The total revenue function is  $q^2 + y^2 = 26q^3$