Example: Solve $\frac{dy}{dx} = \frac{x^2 + y^2}{xy}$

Can we get it in F($\frac{y}{x}$) style?

Start with: $\frac{x^2 + y^2}{xy}$

Separate terms: $\frac{x^2}{xy} + \frac{y^2}{xy}$

Simplify: $\frac{x}{y} + \frac{y}{x}$

Reciprocal of first term: $\left(\frac{y}{x}\right)^{-1} + \frac{y}{x}$

Yes, we have a function of $\frac{y}{x}$.

So let's go:

Start with: $\frac{dy}{dx} = \left(\frac{y}{x}\right)^{-1} + \frac{y}{x}$

$y = vx$ and **$\frac{dy}{dx} = v + x\frac{dv}{dx}$**: $v + x\frac{dv}{dx} = v^{-1} + v$

Subtract v from both sides: $x\frac{dv}{dx} = v^{-1}$

Now use Separation of Variables :

Separate the variables: $v\,dv = \frac{1}{x}\,dx$

Put the integral sign in front: $\int v\,dv = \int \frac{1}{x}\,dx$

Integrate: $\frac{v^2}{2} = \ln(x) + C$

Then we make **$C = \ln(k)$**: $\frac{v^2}{2} = \ln(x) + \ln(k)$

Combine ln: $\frac{v^2}{2} = \ln(kx)$

Simplify: $v = \pm\sqrt{(2\ln(kx))}$

Now substitute back $v = \frac{y}{x}$

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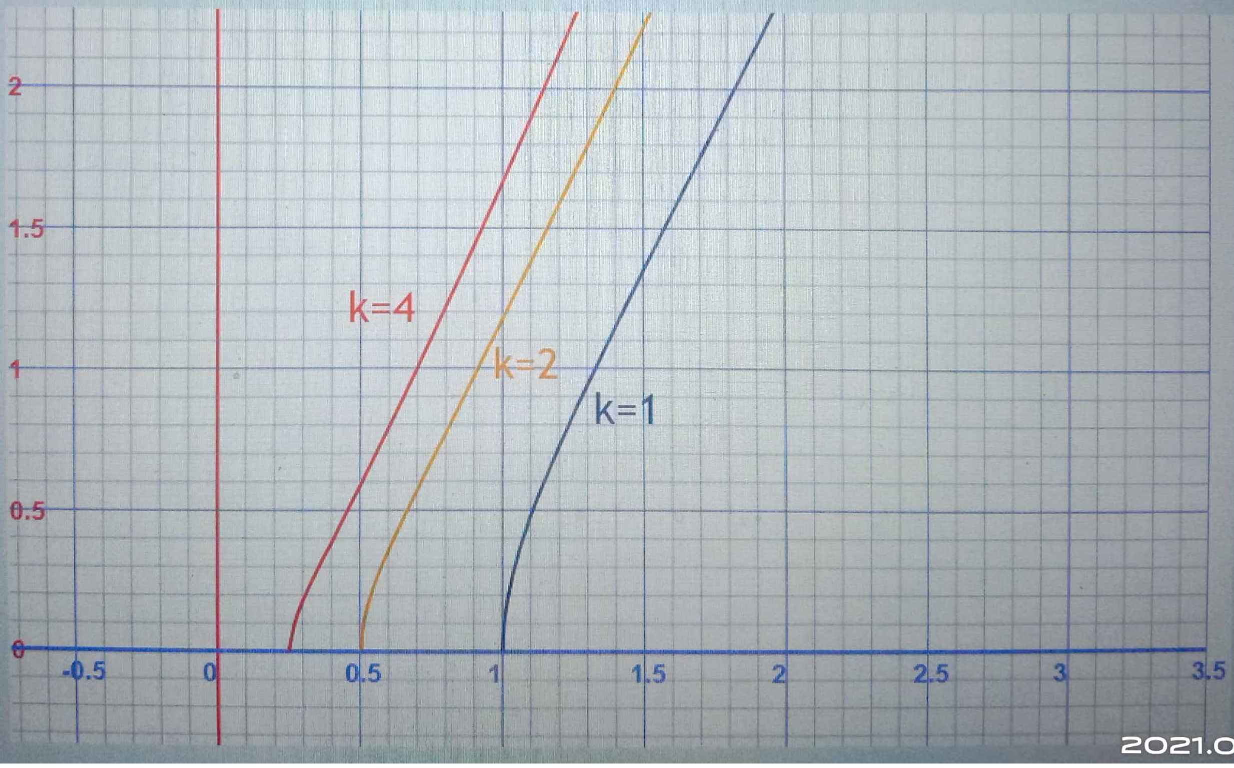
Substitute $v = \frac{y}{x}$: $\frac{y}{x} = \pm\sqrt{(2 \ln(kx))}$

Simplify: $y = \pm x \sqrt{(2 \ln(kx))}$

And we have the solution.

The positive portion looks like this:



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