Example: Solve $\frac{dy}{dx} = \frac{y(x-y)}{x^2}$

Can we get it in $F(\frac{y}{x})$ style?

Start with: $\frac{y(x-y)}{x^2}$

Separate terms: $\frac{xy}{x^2} - \frac{y^2}{x^2}$

Simplify: $\frac{y}{x} - (\frac{y}{x})^2$

Yes! So let's go:

Start with: $\frac{dy}{dx} = \frac{y}{x} - (\frac{y}{x})^2$

$\mathbf{y = vx}$ and $\mathbf{\frac{dy}{dx} = v + x\frac{dv}{dx}}$ $\quad v + x\frac{dv}{dx} = v - v^2$

Subtract v from both sides: $x\frac{dv}{dx} = -v^2$

Now use Separation of Variables:

Separate the variables: $-\frac{1}{v^2}\,dv = \frac{1}{x}\,dx$

Put the integral sign in front: $\int -\frac{1}{v^2}\,dv = \int \frac{1}{x}\,dx$

Integrate: $\frac{1}{v} = \ln(x) + C$

Then we make **C = ln(k)**: $\frac{1}{v} = \ln(x) + \ln(k)$

Combine ln: $\frac{1}{v} = \ln(kx)$

Simplify: $v = \frac{1}{\ln(kx)}$

Now substitute back $v = \frac{y}{x}$

Substitute $v = \frac{y}{x}$: $\frac{y}{x} = \frac{1}{\ln(kx)}$

Simplify: $y = \frac{x}{\ln(kx)}$

And we have the solution.

Here are some sample k values: