

Example 3) Solve: $\left\{ x + y \cos \left(\frac{y}{x} \right) \right\} dx = x \cos \left(\frac{y}{x} \right) dy$

Solution 3) $\left\{ x + y \cos \frac{y}{x} \right\} dx = x \cos \left(\frac{y}{x} \right) dy$

OR, $\frac{dx}{x} = \frac{x + y \cos \left(\frac{y}{x} \right)}{x \cos \left(\frac{y}{x} \right)}$ put $y = vx$; then $\frac{dx}{dx} = v + x \frac{dv}{dx} \dots\dots\dots(1)$

From (1), $v + x \frac{dv}{dx} = \frac{x + vx \cos \left(\frac{vx}{x} \right)}{x \cos \left(\frac{vx}{x} \right)} = \frac{1 + v \cos v}{\cos v}$

Or, $v + x \frac{dv}{dx} = \sec v + v$, or, $x \frac{dv}{dx} = \sec v$, or, $\cos v \cdot dv = \frac{dx}{x}$

Integrating both sides, we get $\int \cos v \, dv = \int \frac{dx}{x} + C$

OR, $\sin v = \log |x| + C$, or, $\sin \frac{y}{x} = \log|x| + C$

Which is the required solution of (1) for the homogeneous equation examples?

Example 4) Find the equation to the curve through $(1, 0)$ for which the slope at a