Example 2) Solve: $(x^2 + y^2)$ dx - 2xy = 0, given that y = 0, when x = 1.

Solution 2) We have $(x^2 + y^2)$ dx - 2xy dy = 0 or, $\frac{dy}{dx} = \frac{x^2 + y^2}{2xy}$ ... (1)

Put y = vx; then $\frac{dy}{dx}$ = v + x$\frac{dv}{dx}$

From, (1), v + x $\frac{dy}{dx} = \frac{x^2 + y^2x^2}{2x^2v} = \frac{1 + v^2}{2v}$

Or, $\frac{2v}{1 - v^2}$. dv = $\frac{dx}{x}$ or, - $\left(\frac{-2v}{1 - v^2}\right)$dv = $\frac{dx}{x}$

Integrating - log(1 - v^2) = log |x| - log C

Or, Log $|1 - v^2|$ = - log |x| + log C

Or, log Log $|1 - v^2|$ x = log C or, (1 - v^2)x = C

Or, $\left(1 - \frac{y^2}{x^2}\right)$ x = C, or, x^2 - y^2 = Cx ...(2)

Given y = 0 when x = 1; therefore, from (2), 1 = C

Hence from (2), the required solution is x^2 - y^2 = x