Example 1) Solve (x^2 - xy) dy = (xy + y^2)dx

Solution 1) We have (x^2 - xy) dy = (xy + y^2)dx ... (1)

The differential equation (1) is a homogeneous equation in x and y.

From (1), we have $\frac{dy}{dx} = \frac{xy + y^2}{x^2 - xy}$ .... (2)

Now put y = vx, then $\frac{dy}{dx}$ = v + x. $\frac{dy}{dx}$

From (2), v + x.$\frac{dy}{dx} = \frac{x.vx + v^{2x^2}}{x^2 - x.vx} = \frac{v + v^2}{1 - v}$

Or, x $\frac{dy}{dx} = \frac{v + v^2}{1 - v}$ - v = $\frac{v + v^2 - v + v^2}{1 - v} = \frac{2v^2}{1 - v}$

Or, $\frac{1-v}{2v^2}$ dv = $\frac{dy}{dx}$ or, $\frac{dx}{x} = \frac{1}{2}\left(\frac{1}{v^2} - \frac{1}{v}\right)$dv

Integrating, log x = $\frac{1}{2}\left(-\frac{1}{v} - logv\right)$ + $\frac{1}{2}$log C

Or, 2 Log x = - $\frac{1}{v}$ - logv + log C or, log x^2 + log v - log C = - $\frac{1}{v}$

OR, Log $\left(\frac{vx^2}{C}\right)$ = - $\frac{x}{y}$ $y = vx$ or, $\frac{vx^2}{C}$ e $\frac{x}{y}$, or, xy = Ce - $\frac{x}{y}$

Which is the required general solution of homogeneous equation examples?