A pair of fair dice is rolled together till a sum of either 5 or 7 is obtained. Then the probability that 5 comes before 7 is

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Probability of getting a sum of $5 = \frac{4}{36} = \frac{1}{9} = P(A)$ as favourable cases are $\{(1,4),(4,1),(2,3),(3,2)\}$

Similarly favourable cases of getting a sum of 7 are $\{(1,6),$ $(6,1),(2,5),(5,2),(3,4),(4,3)\} = 6$

$\therefore$ Prob. of getting a sum of $7 = \frac{6}{36} = \frac{1}{6}$

$\therefore$ Prob. of getting a sum of 5 or 7

$$= \frac{1}{6} + \frac{1}{9} = \frac{5}{18}$$ [as events are mutually exclusive.]

$\therefore$ Prob of getting neither a sum of 5 nor of $7 = \frac{1}{6} - \frac{5}{18} = \frac{13}{18}$

Now we get a sum of 5 before a sum of 7 if either we get a sum of 5 in first chance or we get neither a sum of 5 nor of 7 in first chance and a sum of 5 in second chance and so on. Therefore the required prob. is

$$= \frac{1}{9} + \frac{13}{18} \times \frac{1}{9} + \frac{13}{18} \times \frac{13}{18} \times \frac{1}{9} + \ldots \infty = \frac{1/9}{1 - 13/18} = \frac{1}{9} \times \frac{18}{5} = \frac{2}{5}$$