

$P(A \cup B) = P(A \cap B)$ if and only if the relation between $P(A)$ and $P(B)$ is

Given that $P(A \cup B) = P(A \cap B)$

$$\Rightarrow P(A) + P(B) - P(A \cap B) = P(A \cap B)$$

$$\Rightarrow [P(A) - P(A \cap B)] + [P(B) - P(A \cap B)] = 0$$

But $P(A) - P(A \cap B), P(B) - P(A \cap B) \geq 0$

$$[\because P(A \cap B) \leq P(A), P(B)]$$

$$\Rightarrow P(A) - P(A \cap B) = 0 \text{ and } P(B) - P(A \cap B) = 0$$

$[\because$ Sum of two non-negative no's can be zero only when these no's are zeros]

$$\Rightarrow P(A) = P(B) = P(A \cap B)$$

which is the required relationship.