The solution of primitive integral equation $(x^2 + y^2)\,dy = xy\,dx$ is $y = y(x)$. If $y(1) = 1$ and $(x_0) = e$, then x_0 is equal to

(2005S)

(a) $\sqrt{2(e^2 - 1)}$ (b) $\sqrt{2(e^2 + 1)}$

(c) $\sqrt{3}\,e$ (d) $\sqrt{\dfrac{e^2 + 1}{2}}$

(c) The given D.E. is $(x^2 + y^2)dy = xy\,dx$ s.t. $y(1) = 1$ and $y(x_0) = e$

The given eq^n can be written as

$$\frac{dy}{dx} = \frac{xy}{x^2 + y^2}$$

Put $y = vx$, $\therefore\ v + x\frac{dv}{dx} = \frac{v}{1+v^2}$

$$\Rightarrow \quad x\frac{dv}{dx} = \frac{-v^3}{1+v^2} \Rightarrow \int \frac{1+v^2}{v^3}dv + \int \frac{dx}{x} = 0$$

$$\Rightarrow \quad -\frac{1}{2v^2} + \log|v| + \log|x| = C$$

$$\Rightarrow \quad \log y = C + \frac{x^2}{2y^2} \quad \text{(using } v = y/x\text{)}$$

Also, $y(1) = 1 \Rightarrow \log 1 = C + \frac{1}{2} \Rightarrow C = -\frac{1}{2}$

$\therefore \quad \log y = \frac{x^2 - y^2}{2y^2}$, But given $y(x_0) = e$

$$\Rightarrow \quad \log e = \frac{x_0^2 - e^2}{2e^2} \Rightarrow x_0^2 = 3e^2 \Rightarrow x_0 = \sqrt{3}e$$