If $y = y(x)$ and $\dfrac{2 + \sin x}{y + 1}\left(\dfrac{dy}{dx}\right) = -\cos x$, $y(0) = 1$,

then $y\left(\dfrac{\pi}{2}\right)$ equals   *(2004S)*

(a) 1/3   (b) 2/3   (c) −1/3   (d) 1

(a) $\dfrac{dy}{dx}\left(\dfrac{2+\sin x}{1+y}\right) = -\cos x,\ y(0) = 1$

$\Rightarrow \quad \dfrac{dy}{(1+y)} = \dfrac{-\cos x}{2+\sin x}dx$

Integrating both sides

$\Rightarrow \quad \ln(1+y) = -\ln(2+\sin x) + C$

Put $x = 0$ and $y = 1 \Rightarrow \ln(2) = -\ln 2 + C \Rightarrow C = \ln 4$

Put $x = \dfrac{\pi}{2}\ \ \ln(1+y) = -\ln 3 + \ln 4 = \ln\dfrac{4}{3} \Rightarrow y = \dfrac{1}{3}$