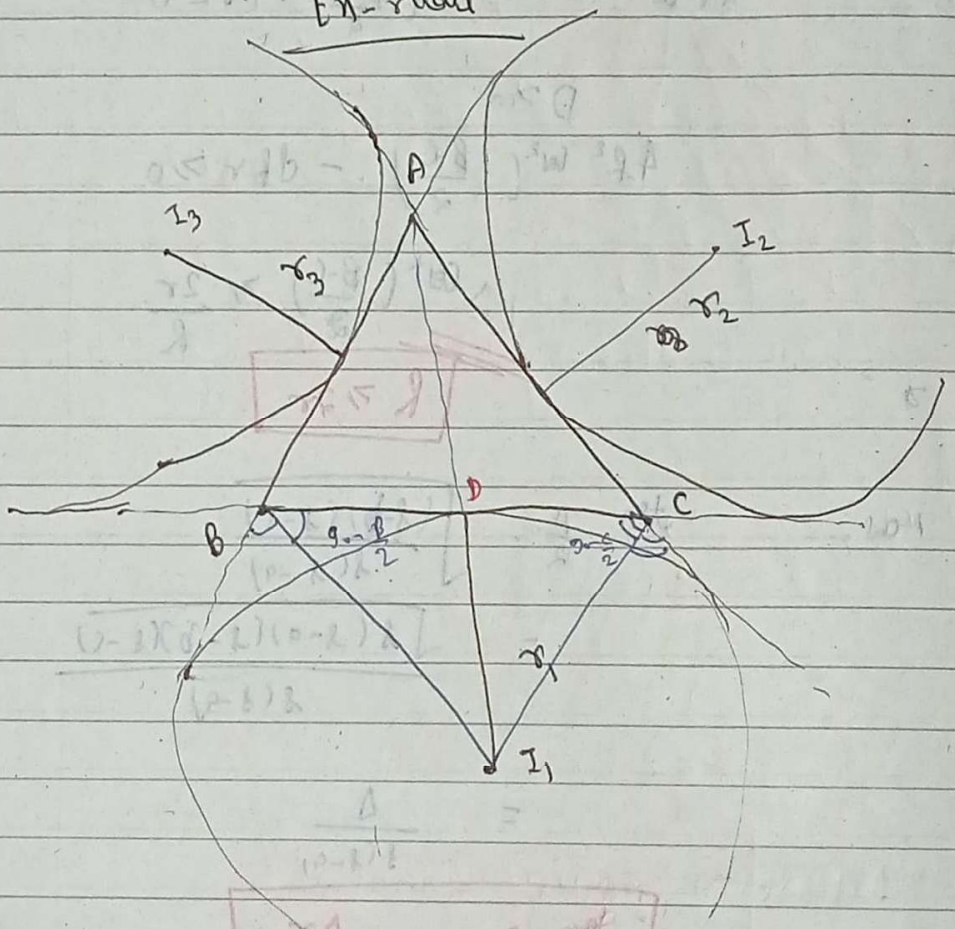
~~Incentre~~

Ex-radii



$$ar(\Delta ABI_1) = ar(\Delta ABI_1 + \Delta A(I_1 - B(I_1)$$

$$= \frac{1}{2} \cdot c \times r_1 + \frac{1}{2} \times b \times r_1 - \frac{1}{2} \times a \times r_1$$

$$\Delta = \frac{(b + (-a)) r_1}{2} = \frac{2(s-a) \cdot r_1}{2}$$

$$\boxed{r_1 = \frac{\Delta}{s-a}} \star\star$$

$$r_2 = \frac{\Delta}{s-b}$$

$$r_3 = \frac{\Delta}{s-c}$$

Note:-

$$\therefore \frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_3} = \frac{s-a+s-b+s-c}{\Delta}$$

$$= \frac{s}{r} = \frac{1}{r}$$

$$\Rightarrow \boxed{\frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_3} = \frac{1}{r}}$$

Now,

$$\cot\left(90^\circ - \frac{B}{2}\right) = \frac{BD}{r_1}$$

$$\cot\left(90^\circ - \frac{C}{2}\right) = \frac{CD}{r_1}$$

$$\tan\frac{B}{2} + \tan\frac{C}{2} = \frac{a}{r_1}$$

$$\Rightarrow r_1 = \frac{a}{2R \sin\frac{A}{2} \cos\frac{B}{2} \cos\frac{C}{2}}$$

$$\boxed{r_1 = 4R \sin\frac{A}{2} \cos\frac{B}{2} \cos\frac{C}{2}}$$