

Question 4

Question. If $A = 75^\circ$, $B = 45^\circ$, then prove that $b + c\sqrt{2} = 2a$.

Solution. $A = 75^\circ$, $B = 45^\circ \Rightarrow C = 60^\circ$

Now, according to Sine Law,

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = k$$

$$\Rightarrow a = k \sin 75^\circ, \quad b = k \sin 45^\circ, \quad c = k \sin 60^\circ$$

$$\Rightarrow \quad \quad \quad b = \frac{k}{\sqrt{2}}, \quad c = \frac{k\sqrt{3}}{2}$$

$$\text{Now, } b + c\sqrt{2} = \frac{k}{\sqrt{2}} + \frac{k\sqrt{3}}{2}$$

$$= \sqrt{2} k \left\{ \frac{1}{2} + \frac{\sqrt{3}}{2} \right\}$$

$$= 2k \left\{ \frac{1}{\sqrt{2}} \cdot \frac{1}{2} + \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{3}}{2} \right\}$$

$$= 2k \left\{ \sin 30^\circ \cdot \cos 45^\circ + \cos 30^\circ \cdot \sin 45^\circ \right\}$$

$$= 2k \sin 75^\circ$$

$$= 2a$$

$$b + c\sqrt{2} = 2a$$

Proved.