Question3

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Question. In ΔABC, $a = 6$, $b = 3$ & $\cos(A-B) = \frac{4}{5}$.
Find area of ΔABC.

(a) 9

(b) 18

(c) 36

(d) 27

Solution. Using Sine Law: →

$$\frac{a}{\sin A} = \frac{b}{\sin B}$$

$$\Rightarrow \sin A = \frac{a}{b} \sin B$$

$$= \frac{6}{3} \sin B$$

$$\sin A = 2 \sin B \quad \text{—— (1)}$$

Given, $\cos(A-B) = \frac{4}{5}$

$$\therefore \cos\left(\frac{A-B}{2}\right) = \sqrt{\frac{1+\cos(A-B)}{2}}$$

$$= \sqrt{\frac{1+(4/5)}{2}}$$

$$= \frac{3}{\sqrt{10}} \quad \text{——— } (2)$$

$$\Rightarrow \tan\left(\frac{A-B}{2}\right) = \frac{\sqrt{1-\cos^2\left(\frac{A-B}{2}\right)}}{\cos\left(\frac{A-B}{2}\right)}$$

$$= \frac{\sqrt{1-\frac{9}{10}}}{\frac{3}{10}}$$

$$\tan\left(\frac{A-B}{2}\right) = \frac{1}{3} \quad \text{——— } (3)$$

Using Napier's Analogy: →

$$\boxed{\tan\left(\frac{A-B}{2}\right) = \frac{a-b}{a+b} \cdot \cot\frac{C}{2}}$$

$$\Rightarrow \quad \frac{1}{3} = \frac{6-3}{9} \times \cot\frac{C}{2}$$

$$\Rightarrow \quad \frac{1}{3} = \frac{6-3}{6+3} \times \cot\frac{C}{2}$$

$$\Rightarrow \quad \cot\frac{C}{2} = 1$$

$$\Rightarrow \quad \frac{C}{2} = 45^\circ$$

$$\Rightarrow \quad C = 90^\circ \quad \text{——— (4)}$$

$\therefore$ Area of $\Delta ABC = \frac{1}{2} ab \sin C$

$$= \frac{1}{2} \times 6 \times 3 \times \sin(90^\circ)$$

$$= 9$$ Ans. option (a)