

Question. Let ABC and ABC' be two non-congruent triangles with sides $AB = 4$, $AC = AC' = 2\sqrt{2}$ and angle $B = 30^\circ$. The absolute value of the difference between the areas of these triangles is: \rightarrow

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Solution.

Let $BC = a$

$$\therefore \cos B = \frac{a^2 + 16 - (2\sqrt{2})^2}{2 \times a \times 4}$$

$$\Rightarrow \frac{\sqrt{3}}{2} = \frac{a^2 + d}{4a}$$

$$\Rightarrow 4\sqrt{3}a = a^2 + d$$

$$\Rightarrow a^2 - 4\sqrt{3}a + d = 0$$

$$\Rightarrow a_1 + a_2 = 4\sqrt{3}, \quad a_1 a_2 = d$$

$$\text{Now, } |\Delta_1 - \Delta_2| = \left| \frac{1}{2} \times a_1 \times 4 \times \sin B - \frac{1}{2} \times a_2 \times 4 \times \sin B \right|$$

$$= 2 \times \sin 30^\circ |a_1 - a_2|$$

$$= \sqrt{(a_1 + a_2)^2 - 4a_1 a_2}$$

$$= \sqrt{16 - 4d}$$

$$= \sqrt{48 - 4 \times 8}$$

$$= 4$$

⇒

$$|\Delta_1 - \Delta_2| = 4 \text{ sq. units} \quad \underline{\underline{\text{Ans.}}}$$