PYQ3

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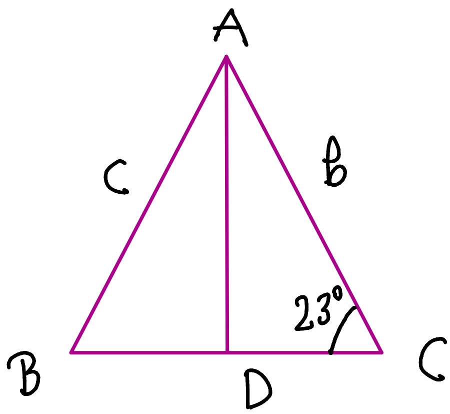
Question. In the ΔABC, AD is the altitude from A. Given $b > c$, $\angle C = 23^\circ$ and $AD = \frac{abc}{b^2 - c^2}$, then $\angle B$ is equal to

[Integer Type IIT-JEE 1994,2]

Solution



Using Cosine law: $\rightarrow$

$$\cos B = \frac{a^2 + c^2 - b^2}{2ac}$$

$$= \frac{a^2}{2ac} - \left(\frac{b^2 - c^2}{abc}\right) \cdot \frac{b}{2} \quad \text{—— (1)}$$

Now, we know that Length of Altitude

$$\boxed{AD = \frac{abc}{b^2 - c^2}}$$

$\therefore$ From equation (1): $\rightarrow$

$$\cos B = \frac{a}{2c} - \frac{b}{2AD} \quad \text{—— (2)}$$

In ΔACD, $\sin C = \frac{AD}{b}$ —— (3)

Using Sine Law for ΔABC,

$$\frac{a}{\sin A} = \frac{c}{\sin C}$$

$$\Rightarrow \quad \frac{a}{c} = \frac{\sin A}{\sin C} \quad \text{—— (4)}$$

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$\therefore$ Rewriting equation (2) using values from equations (3) & (4) :→

$$\cos B = \frac{\sin A}{2\sin C} - \frac{1}{2\sin C}$$

$$\Rightarrow \quad 2\sin C \cos B = \sin[\pi - (B+C)] - 1$$

$$\Rightarrow \quad \sin(B+C) - \sin(B-C) = \cancel{\sin(B+C)} - 1$$

$$\Rightarrow \quad \sin(B-C) = 1$$

$$\Rightarrow \quad B = 90^\circ + C$$

$$\Rightarrow \quad \boxed{B = 113^\circ}$$

Ans.