PYQ1

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Question. ABCD is a trapezium such that AB and CD are parallel and $BC \perp CD$. If $\angle ADB = \theta$, $BC = p$ and $CD = q$, then AB is equal to:→

[JEE Mains 2013]

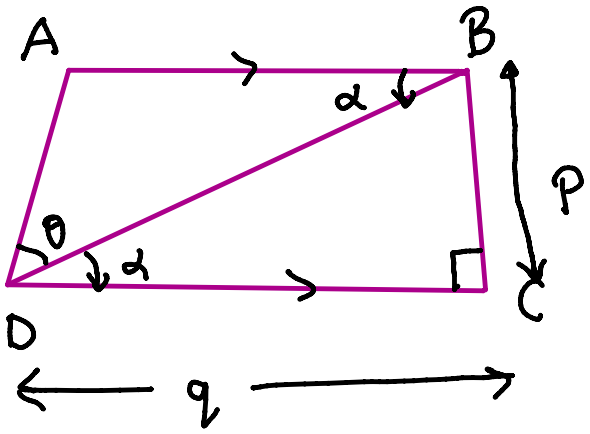
(a) $\dfrac{p^2 + q^2\cos\theta}{p\cos\theta + q\sin\theta}$

(b) $\dfrac{p^2 + q^2}{p^2\cos\theta + q^2\sin\theta}$

(c) $\dfrac{(p^2 + q^2)\sin\theta}{(p\cos\theta + q\sin\theta)^2}$

(d) $\dfrac{(p^2 + q^2)\sin\theta}{p\cos\theta + q\sin\theta}$

Solution.



Let $\angle BDC = \alpha$

In $\triangle BCD$,

$BD = \sqrt{BC^2 + CD^2}$

$= \sqrt{p^2 + q^2}$ — (1)

also, In $\triangle ABD$, $\angle DAB = \pi - (\theta + \alpha)$

Also, In ΔABD, $\angle DAB = \pi - (\theta + \alpha)$

Now, using Sine rule in ΔABD,

$$\frac{AB}{\sin\theta} = \frac{BD}{\sin\{\pi - (\theta+\alpha)\}}$$

$$\Rightarrow \quad AB = \frac{\sin\theta}{\sin(\theta+\alpha)} \cdot BD$$

$$= \frac{\sin\theta}{\sin\theta\cdot\cos\alpha + \cos\theta\cdot\sin\alpha} \cdot \sqrt{p^2+q^2}$$

$$= \frac{\sin\theta}{\frac{\sin\theta\cdot q}{\sqrt{p^2+q^2}} + \frac{\cos\theta\cdot p}{\sqrt{p^2+q^2}}} \times \sqrt{p^2+q^2}$$

$$\Rightarrow \quad AB = \frac{(p^2+q^2)\sin\theta}{p\cos\theta + q\sin\theta}$$

Ans.

Option (d)