

Evaluate $\int \cos 2\theta \log_e \left(\frac{\cos \theta + \sin \theta}{\cos \theta - \sin \theta} \right) d\theta$

$$\begin{aligned} I &= \int \cos 2\theta \log_e \left(\frac{\cos \theta + \sin \theta}{\cos \theta - \sin \theta} \right) d\theta \\ &= \log_e \left(\frac{\cos \theta + \sin \theta}{\cos \theta - \sin \theta} \right) \int \cos 2\theta d\theta - \int \left(\left[\log_e \left(\frac{\cos \theta + \sin \theta}{\cos \theta - \sin \theta} \right) \right]' \int \cos 2\theta d\theta \right) d\theta \\ &= \frac{\sin 2\theta}{2} \log_e \left(\frac{\cos \theta + \sin \theta}{\cos \theta - \sin \theta} \right) \\ &\quad - \int \frac{\sin 2\theta (\cos \theta - \sin \theta)}{2(\sin \theta + \cos \theta)} \cdot \frac{2}{(\cos \theta - \sin \theta)^2} d\theta \\ &= \frac{\sin 2\theta}{2} \log_e \left(\frac{\cos \theta + \sin \theta}{\cos \theta - \sin \theta} \right) - \int \tan 2\theta d\theta \\ I &= \sin 2\theta \log_e \sqrt{\frac{\sin \theta + \cos \theta}{\cos \theta - \sin \theta}} - \frac{1}{2} \log_e |\sec 2\theta| + c \end{aligned}$$