Evaluate $\int \frac{dx}{\cos x\sqrt{\cos 2x}}$.

$$
\begin{aligned}
I &= \int \frac{dx}{\cos x\sqrt{\cos 2x}} \\
&= \int \frac{\cos x dx}{\cos^2 x\sqrt{1-2\sin^2 x}} \\
&= \int \frac{\cos x dx}{(1-\sin^2 x)\sqrt{1-2\sin^2 x}} \\
&= \int \frac{dt}{(1-t^2)\sqrt{1-2t^2}}
\end{aligned}
$$

Put $t = \frac{1}{y}$

$$
\begin{aligned}
\therefore\ dt &= -\frac{1}{y^2}dy \\
\therefore\ I &= -\int \frac{dy}{y^2\left(1-\frac{1}{y^2}\right)\sqrt{1-\frac{2}{y^2}}} \\
&= -\int \frac{ydy}{(y^2-1)\sqrt{y^2-2}}
\end{aligned}
$$

Now put $y^2 - 2 = z^2$

$$
\begin{aligned}
\text{then, } ydy &= zdz \\
\therefore\ I &= -\int \frac{dz}{z^2+1} \\
&= -\tan^{-1} z + c \\
&= -\tan^{-1} \sqrt{y^2-2} + c \\
&= -\tan^{-1} \sqrt{\frac{1}{t^2}-2} + c \\
&= -\tan^{-1} \sqrt{\frac{1}{\sin^2 x}-2} + c
\end{aligned}
$$