Q) $\int \frac{xdx}{x^4 + x^2 + 1}$ equals

(a) $\frac{2}{3}\tan^{-1}\left(\frac{2x^2+1}{\sqrt{3}}\right) + C$

(b) $\frac{1}{\sqrt{3}}\left[\tan^{-1}\left(\frac{2x-1}{\sqrt{3}}\right) - \tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)\right] + C$

(c) $\frac{1}{\sqrt{3}}\tan^{-1}\left(\frac{2x^2+1}{\sqrt{3}}\right) + C$

(d) $\frac{1}{\sqrt{3}}\left[\tan^{-1}\left(\frac{2x-1}{\sqrt{3}}\right) + \tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)\right] + C$

where C is an arbitrary constant.

Solution: I

$$= \int \frac{xdx}{(x^4+2x^2+1)-x^2}$$

$$= \int \frac{xdx}{(x^2+x+1)(x^2-x+1)}$$

$$= \frac{1}{2}\int \frac{(x^2+x+1)-(x^2-x+1)}{(x^2+x+1)(x^2-x+1)}dx$$

$$= \frac{1}{2}\int \frac{dx}{x^2-x+1} - \frac{1}{2}\int \frac{dx}{x^2+x+1}$$

$$= \frac{1}{2}\int \frac{dx}{(x-(1/2))^2-(\sqrt{3}/2)^2} - \frac{1}{2}\int \frac{dx}{(x+(1/2))^2-(\sqrt{3}/2)^2}$$

$$= \frac{1}{\sqrt{3}}\tan^{-1}\left(\frac{2x-1}{\sqrt{3}}\right) - \frac{1}{\sqrt{3}}\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)$$

$$= \frac{1}{\sqrt{3}}\left[\tan^{-1}\left(\frac{2x-1}{\sqrt{3}}\right) - \tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)\right] + C$$

Answer is option B