

Evaluate: $\int \sqrt{\frac{1-\sqrt{x}}{1+\sqrt{x}}} dx.$

Answer: $\int \sqrt{\frac{1}{1+\sqrt{x}}} dx$

$$= \int \sqrt{\frac{1-\sqrt{x}}{1+\sqrt{x}}} \cdot \frac{1-\sqrt{x}}{1-\sqrt{x}} dx$$

$$= \int \frac{(1+\frac{1}{x^2})dx}{x^2+1+\frac{1}{x^2}} - \int \frac{dt}{t^2+t+1}, \text{ where } t = x^2$$

$$= \int \frac{(1+\frac{1}{x^2})dx}{(x-\frac{1}{x})^2+3} - \int \frac{dt}{(t+\frac{1}{2})^2+\frac{3}{4}}$$

$$= \frac{1}{\sqrt{3}} \tan^{-1} \frac{x-\frac{1}{x}}{\sqrt{3}} - \frac{1}{\sqrt{3}} \tan^{-1} \frac{t+\frac{1}{2}}{\frac{\sqrt{3}}{2}} + c$$

$$= \frac{1}{\sqrt{3}} \tan^{-1} \frac{x^2-1}{\sqrt{3}x} - \frac{2}{\sqrt{3}} \tan^{-1} \frac{2x^2+1}{\sqrt{3}} + c$$