* Homogeneous of D.E :-

$$\frac{dy}{dx} = \frac{f(x,y)}{g(x,y)}$$

$$f(tx, ty) = t^n f(x,y)$$

$$y/x = v$$

$$y = xv$$

$$\frac{dy}{dx} = x\frac{dv}{dx} + v$$

$$\boxed{\frac{dy}{dx} = \phi\left(y/x\right)}$$

(1) $\frac{dy}{dx} = \frac{x+2y}{x}$

$$\frac{dy}{dx} = x + 2\left(y/x\right)$$

$$y/x = v$$

$y = vx$

$\frac{dy}{dx} = v + x\frac{dv}{dx}$

$v + x\frac{dv}{dx} = 1 + 2v$

$x\frac{dv}{dx} = 1 + v$

$\int \frac{dv}{1+v} = \int \frac{dx}{x}$

$\ln|1+v| = \ln|x| + \ln c$

$1 + v = cx$

$1 + y/x = cx$

$(x-y)\frac{dy}{dx} = x + 2y$

$\frac{dy}{dx} = \frac{x+2y}{x-y}$

$\frac{dy}{dx} = \frac{1+2y/x}{1-y/x}$

$v + x\frac{dv}{dx} = \frac{1+2v}{1-v}$

$x\frac{dv}{dx} = \frac{1+2v}{1-v} - v$

$$x\frac{dv}{dx} = \frac{1+v+v^2}{1-v}$$

$$\int \frac{1-v}{1+v+v^2}\,dv = \int \frac{dx}{x}$$

$$1-v = A(2v+1)+B$$

$$-1 = 2A \quad A = -1/2$$

$$A+B = 1$$

$$B = 3/2$$

$$\int \frac{-\frac{1}{2}(2v+1)}{1+v+v^2} + \int \frac{3/2}{1+v+v^2}\,dv = \int \frac{dx}{x}$$

$$-\frac{1}{2}\ln(1+v+v^2) + \frac{3}{2}\int \frac{dv}{(v+1/2)^2+3/4} = \ln|x|$$

$$-\frac{1}{2}\ln(1+v+v^2) + \frac{3}{2}\left(\frac{4}{3}\right)\tan^{-1}\left(\frac{v+1/2}{3/4}\right) = $$

(3) $$x\cos\left(y/x\right)\frac{dy}{dx} = y\cos\left(y/x\right)+x$$

$$y/x = v$$

$$\frac{dy}{dx} = \frac{y\cos(y/x)+x}{x\cos(y/x)}$$

$$\frac{dy}{dx} = y/x + \sec(y/x)$$

$$\cancel{v} + x\frac{du}{dx} = \cancel{v} + \sec u$$

$$\frac{du}{\sec u} = \frac{dx}{x}$$

$$\int \cos u \, du = \int \frac{dx}{x}$$

$$\sin u = \ln|x| + c$$

$$u = y/x$$

(4)

$$x^2\frac{dy}{dx} = x^2 - 2y^2 + xy$$

$$\frac{dy}{dx} = 1 - 2\left(y/x\right)^2 + y/x$$

$$y/x = v$$

$$\frac{dy}{dx} = v + x\frac{dv}{dx}$$

$$\cancel{v} + x\frac{dv}{dx} = 1 - 2v^2 + \cancel{v}$$

$$\int \frac{dv}{(1-\sqrt{2}v)(1+\sqrt{2}v)} = \int \frac{dx}{x}$$

$$\frac{1}{2}\int \frac{(1-\sqrt{2}v) + (1+\sqrt{2}v)}{(1-\sqrt{2}v)(1+\sqrt{2}v)} dv$$

$$\frac{1}{2}\int \left(\frac{1}{1+\sqrt{2}v} + \frac{1}{1-\sqrt{2}v}\right) dv$$

$y/x = v$

$$v + x\frac{dv}{dx} = \frac{1+v}{2v}$$

$$x\frac{dv}{dx} = \frac{1+v}{2v} - v$$

$$x\frac{dv}{dx} = \frac{1+v-2v^2}{2v}$$

$$\int \frac{2v\,dv}{1+v-2v^2} = \int \frac{dx}{x}$$

$$\int \frac{\cancel{2v+1}\,dv}{(1-v)(1+2v)} = \int \frac{dx}{x}$$

$$\Rightarrow \int \frac{-2v\,dv}{(v-1)(2v+1)} = \int \frac{dx}{x}$$

$$\Rightarrow \int \left(\frac{-2/3}{v-1} + \frac{-2/3}{2v+1}\right) dv = \int \frac{dx}{x}$$

$$\Rightarrow \int -\frac{2}{3}\ln(v-1) - \frac{2}{3}\ln(2v+1) = \ln x + \ln c$$

$$\Rightarrow -\frac{2}{3}\left(\ln (v-1)(2v+1)\right) = \ln x + \ln c$$

$$= -\frac{2}{3}\left(\ln\left(\frac{y}{x}-1\right)\right) \frac{2}{3}\ln\left(\frac{2y}{x}-1\right) = \ln x \frac{2}{3} \ln c$$

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(16) Find the curve such that the ordinate of any of its points is the proportional mean b/w the abscissa and the sum of the abscissa and subnormal at the point.

$\Rightarrow$

$$y = \sqrt{x\left(x + y\frac{dy}{dx}\right)}$$

$$y^2 = x^2 + xy\frac{dy}{dx}$$

$$\frac{dy}{dx} = \frac{x^2 - y^2}{xy}$$

$$\frac{dy}{dx} = \frac{1 - (y/x)^2}{(y/x)}$$

$$u + x\frac{du}{dx} = \frac{1 - u^2}{u}$$

$$x\frac{du}{dx} = \frac{1 - 2u^2}{u}$$

$$\int \frac{u\,du}{\cancel{1} - 2u^2} = \int \frac{x}{dx}$$

$$\int \frac{u}{(\sqrt{2}v + 1)(\sqrt{2}v - 1)}du = -\int \frac{dx}{x}$$

$$\int \left(\frac{\frac{1}{2\sqrt{2}}}{(\sqrt{2} + 1)} + \frac{\frac{1}{2\sqrt{2}}}{(\sqrt{2}v - 1)}\right) du = \int \frac{dx}{x}$$

$$\frac{1}{2\sqrt{2}} \ln\left(\frac{\sqrt{2}v+1}{\sqrt{2}}\right) + \frac{1}{2\sqrt{2}} \ln\left(\frac{\sqrt{2}v-1}{\sqrt{2}}\right) = -\ln x + \ln c$$

$$\frac{1}{2\sqrt{2}} \ln\left(\sqrt{2}\right.$$

$$\frac{1}{4}\left[\ln\left(\sqrt{2}\,x/y+1\right) + \ln\left(\sqrt{2}\,x/y-1\right) = -\ln y + \ldots\right.$$

Q(11) Find the curve such that the angle, formed with the x-axis by the tangent to the curve at any of its points, is twice the angle formed by the polar radius of the point of tangency with the x-axis.

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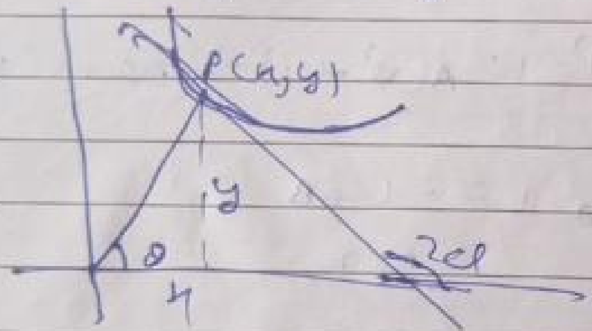

$$\frac{dy}{dx} = \tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta}$$

$$\tan\theta = y/x$$

$$\frac{dy}{dx} = \frac{2\,y/x}{1-(y/x)^2}$$

$$y/x = v$$

$$v + x\frac{dv}{dx} = \frac{2v}{1-v^2}$$

$$x\frac{dv}{dx} = \frac{2v}{1-v^2} - v$$

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$$x\frac{dv}{dx} = \frac{v+v^3}{1-v^2}$$

$$x\frac{dv}{dx} = \frac{v(1+v^2)}{(1-v^2)}$$

$$\int \frac{(v^2-1)\,dv}{\cancel{1+v^2}\,v(v^2+1)} = -\int \frac{dx}{x}$$

$$\frac{v^2-1}{v(v^2+1)} = \frac{A}{v} + \frac{Bv+C}{v^2+1}$$

$$A = -1$$

$$v^2-1 = A(v^2+1) + (Bv+C)v$$

$$1 = -1 + B$$
$$B = 2$$

$$0 = C$$

$$I = \int\left(\frac{-1}{v} + \frac{2v}{v^2+1}\right)dv = -\ln x + c$$

$$-\ln v + \ln(v^2+1) = -\ln x + c$$

$$-\ln(y/x) + \ln((y/x)^2+1) = -\ln x + c$$

(12) find the curve such that the ratio of the subnormal at any point to the sum of its abscissa and ordinates is equal to the ratio of the ordinates is equal to the ratio of the ordinate of this point to its

abscissa of the curve passes through (1,0), find all possible eqⁿ in form $y=f(x)$.

$\Rightarrow$ $\dfrac{\left|y\frac{dy}{dx}\right|}{x+y} = y/x$

$$y\frac{dy}{dx} = \frac{y(x+y)}{x}$$

$$\frac{dy}{dx} = \pm \frac{y(x+y)}{xy}$$

$$\frac{dy}{dx} = \pm \frac{\left(\frac{y}{x}\right) + \left(\frac{y}{x}\right)^2}{\left(y/x\right)}$$

$$y/x = u$$

$$u + x\frac{du}{dx} = \pm \frac{u+u^2}{u}$$

$$x\frac{du}{dx} = \pm \frac{u+u^2}{u} - u$$

$$x\frac{du}{dx} = \frac{\cancel{u}}{\cancel{u}} \quad (1)$$

$$\int du = \int \frac{dx}{x}$$

$$u = \ln|x| + c$$

$$y/x = \ln|x| + c$$

$y = x\ln x + c$

$0 = 0 + c$

$c = 0$

$y = x\ln x$

$v + x\frac{dv}{dx} = \frac{-(v+v^2)}{v}$

$x\frac{dv}{dx} = -1 - v - v$

$x\frac{dv}{dx} = -2v - 1$

$\int \frac{dv}{2v+1} = -\int \frac{dx}{x}$

$\frac{\ln(2v+1)}{2} = -\ln x + c$

$\ln\left(2\frac{y}{x} + 1\right) = -2\ln x + c_1$

$0 = 0 + c_1$

$c_1 = 0$

$\ln\left(2\frac{y}{x} + 1\right) = \ln\left(\frac{1}{x^2}\right)$

$\frac{2y + x}{x} = \frac{1}{x^2}$

$2y = \frac{1}{x} - x$

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$$y = \frac{1}{2}\left(\frac{1-x^2}{x}\right)$$

Q(13) Find the curve for which the sum of the normal and subnormal is proportional to the abscissa, the proportionality constant being 1.

$$\left| y\sqrt{1+\left(\frac{dy}{dx}\right)^2} \right| + \left| y\frac{dy}{dx} \right| = x$$

$$y\sqrt{1+m^2} = x - |ym|$$

$$y^2(1+m^2) = x^2 + y^2m^2 - 2x|my|$$

$$|my| = \frac{x^2-y^2}{2x}$$

$$\frac{dy}{dx} = \pm\frac{x^2-y^2}{2xy}$$

(+)

$$\frac{dy}{dx} = \frac{x^2-y^2}{2xy}$$

$$\frac{dy}{dx} = \frac{1-(y/x)^2}{2(y/x)}$$

$$v + x\frac{dv}{dx} = \frac{1-v^2}{2v}$$

$$x\frac{dv}{dx} = \frac{1-3v^2}{2v}$$

$$\frac{2v\,dv}{1-3v^2} = \frac{dx}{x}$$

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$$\int \frac{2v\,dv}{3v^2-1} = -\int \frac{dx}{x}$$

$$\frac{1}{3}\int \frac{6v}{3v^2-1}\,dv = -\int \frac{dx}{x}$$

$$\frac{1}{3}\ln(3v^2-1) = -\ln x + c$$

$$\frac{1}{3}\ln\left(3y^2/x^2-1\right) = -\ln x + c$$

(14) The area of the figure bounded by a curve, the x-axis and two ordinates, one of which is constant, the other is variable is equal to the ratio of the cube of the variable ordinate to the variable ordinate to the variable abscissa, find the eqn of curve if it passes through (1,0)

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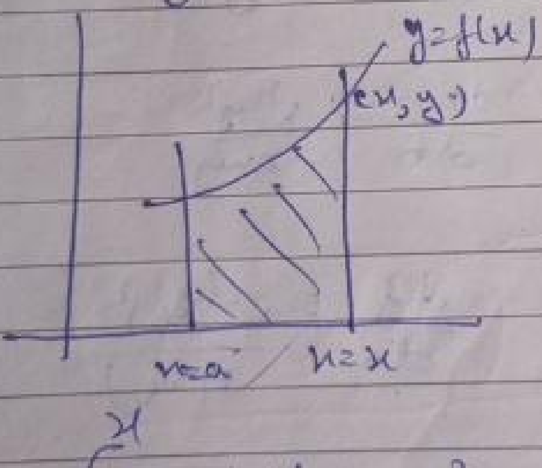

$$\int_a^x f(t)\,dt = y^3/x$$

$$f(x) = \frac{x \cdot 3y^2\, dy/dx - y^3}{x^2} = y$$

$$\frac{dy}{dx} = \frac{x^2 y + y^3}{3xy^2}$$

$$\frac{dy}{dx} = \frac{y/x + (y/x)^3}{3y^2/x^2}$$

$y/x = v$

$$v + x\frac{dv}{dx} = \frac{v + v^3}{3v^2}$$

$$v + x\frac{dv}{dx} = \frac{1+v^2}{3v}$$

$$x\frac{dv}{dx} = \frac{1-2v^2}{3v}$$

$$\int \frac{3v\,dv}{2v^2-1} = -\int \frac{dx}{x}$$

$$\frac{3}{4}\int \frac{4v}{2v^2-1}\,dv = -\int \frac{dx}{x}$$

$$\frac{3}{4}\ln(2v^2-1) = -\ln x + C$$

$$\frac{3}{4}\ln\left(2(y/x)^2 - 1\right) = -\ln x + C$$

Eq^n reducible to homogeneous eq^n:-

$$\frac{dy}{dx} = \frac{a_1x + b_1y + c_1}{a_2x + b_2y + c_2} \begin{matrix} \to L_1 \\ \to L_2 \end{matrix}$$

i) $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

ii) $a_2 + b_1 \neq 0$