Evaluate $\int x^{-1/2}\left(2+3x^{1/3}\right)^{-2}dx$

$I=\int x^{-1/2}(2+3x^{1/3})^{-2}dx$
Let $x=t^6$ or $dx=6t^5dt$

$$\therefore\ I \quad =\int\ t^{-3}(2+3t^2)^{-2}\cdot 6t^5dt$$
$$=6\int\ \frac{t^2}{(2+3t^2)^2}dt$$
$$=\frac{6}{9}\int\ \frac{t^2dt}{\left(\frac{2}{3}+t^2\right)^2}$$

Now, let $t=\sqrt{\left(\frac{2}{3}\right)}\tan\ \theta$

$$\therefore\ dt=\sqrt{\left(\frac{2}{3}\right)}\sec^2\ \theta d\theta$$

$$\therefore\ I=\frac{6}{9}\int\frac{\frac{2}{3}\tan^2\ \theta\cdot\sqrt{\left(\frac{2}{3}\right)}\sec^2\ \theta d\theta}{\frac{4}{9}\sec^4\ \theta}$$

$$=\sqrt{\frac{2}{3}}\int\sin^2\ \theta d\theta$$
$$=\frac{1}{\sqrt{6}}\int(1-\cos\ 2\theta)d\theta$$
$$=\frac{1}{\sqrt{6}}\left\{\theta-\frac{\sin\ 2\theta}{2}\right\}+c$$
$$=\frac{1}{\sqrt{6}}\left\{\theta-\frac{\tan\ \theta}{1+\tan^2\ \theta}\right\}+c$$
$$=\frac{1}{\sqrt{6}}\left\{\tan^{-1}\left\{\sqrt{\frac{3}{2}}t\right\}-\frac{\sqrt{\frac{3}{2}}\cdot t}{1+\frac{3}{2}t^2}\right\}+c$$
$$=\frac{1}{\sqrt{6}}\left\{\tan^{-1}\left\{\sqrt{\frac{3}{2}}x^{1/6}\right\}-\frac{\sqrt{6}x^{1/6}}{2+3x^{1/3}}\right\}+c$$