Evaluate $\int (x-5)\sqrt{x^2+x}dx$

ANSWER: Let $(x-5) = \lambda \frac{d}{dx}(x^2+x) + \mu$.
Then, $x - 5 = \lambda(2x+1) + \mu$
Comparing coefficients of like power of x, we get

$1 = 2\lambda$ and $\lambda + \mu = -5$or $\lambda = \frac{1}{2}$ and $\mu = -\frac{11}{2}$. Therefore,

$$\int (x-5)\sqrt{x^2+x}dx = \int \left[\frac{1}{2}(2x+1) - \frac{11}{2}\right]\sqrt{x^2+x}dx$$

$$= \frac{1}{2}\int (2x+1)\sqrt{x^2+x}dx - \frac{11}{2}\int \sqrt{x^2+x}dx$$

$$= \frac{1}{2}\int \sqrt{t}dt - \frac{11}{2}\int \sqrt{\left(x+\frac{1}{2}\right)^2 - \left(\frac{1}{2}\right)^2}\, dx,$$

$$= \frac{1}{2}\frac{t^{3/2}}{3/2} - \frac{11}{2}\left[\frac{1}{2}\left(x+\frac{1}{2}\right)\sqrt{\left(x+\frac{1}{2}\right)^2 - \left(\frac{1}{2}\right)^2}\, dx + \sqrt{\left(x+\frac{1}{2}\right)^2 - \left(\frac{1}{2}\right)^2}\right] + C$$

$$-\frac{1}{2}\left(\frac{1}{2}\right)^2 \log | \left(x+\frac{1}{2}\right) + \sqrt{(x)} + x$$

$$= \frac{(x^2+x)^{3/2}}{3} - \frac{11}{2}\left[\frac{2x+1}{4}\sqrt{x^2+x} - \frac{1}{8}\log\left|\left(x+\frac{1}{2}\right) + \sqrt{x^2+x}\right|\right] + C$$