Evaluate $\int \sqrt{\left(\frac{1+x^2}{x^2-x^4}\right)}\,dx$

Answer: $\int \sqrt{\left(\frac{1+x^2}{x^2-x^4}\right)}\,dx = \int \frac{1+x^2}{x\sqrt{(1-x^4)}}\,dx$

$$= \int \frac{dx}{x\sqrt{(1-x^4)}} + \int \frac{xdx}{\sqrt{(1-x^4)}}$$
$$= \int \frac{x^3dx}{x^4\sqrt{(1-x^4)}} + \int \frac{xdx}{\sqrt{(1-x^4)}}$$
$$= -\frac{1}{2}\int \frac{udu}{(1-u^2)u} + \frac{1}{2}\int \frac{dv}{\sqrt{(1-v^2)}}$$

(Putting $1 - x^4 = u^2, -4x^3dx = 2$)
in the first integral and $x^2 = 2xdx = dv$ in the second inerert-

$$= \frac{1}{2}\int \frac{du}{u^2-1} + \frac{1}{2}\sin^{-1}\ v$$
$$= \frac{1}{2}\cdot\frac{1}{2\times1}\log\left|\frac{u-1}{u+1}\right| + \frac{1}{2}\sin^{-1}\ x + c$$
$$= \frac{1}{4}\log\left|\frac{\sqrt{(1-x^4)}-1}{\sqrt{(1-x^4)}+1}\right| + \frac{1}{2}\sin^{-1}\ (x^2) + c$$