PYQ5

Question. In a non-right angled triangle PQR, let p, q and r denote the lengths of the sides opposite to the angles at P, Q, R respectively. The median from R meets the side PQ at S, the perpendicular from P meets the side QR at E, and RS and PE intersect at O. If $p = \sqrt{3}$, $q = 1$, and the radius of the circumcircle of the ΔPQR equals 1, then which of the following option is/are correct?

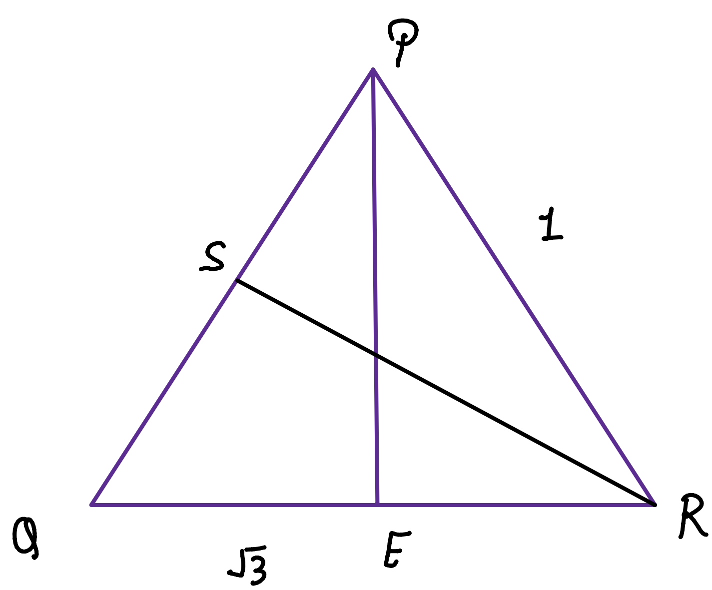
(A) Length of $OE = \frac{1}{6}$ [JEE Advanced 2019]

(B) Radius of incircle of $\Delta PQR = \frac{\sqrt{3}}{2}(2-\sqrt{3})$

(C) Length of $RS = \frac{\sqrt{7}}{2}$

(d) Area of $\Delta SOE = \frac{\sqrt{3}}{12}$

Solution.



By Sine Rule,

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$$\boxed{\frac{p}{\sin P} = \frac{q}{\sin Q} = \frac{r}{\sin R} = 2R \text{ (Circumradius)}} \longrightarrow (1)$$

$\therefore \quad \sin P = \dfrac{p}{2R}, \quad \sin Q = \dfrac{q}{2R}$

$\Rightarrow \sin P = \dfrac{\sqrt{3}}{2}, \quad \sin Q = \dfrac{1}{2}$

$\Rightarrow P = 60^\circ$ or 120° , $Q = 30^\circ$ or 150°

Case (i):. If $P = 60^\circ$ and $Q = 30^\circ$:-

then $R = 90^\circ$ which is not possible as it is given that the triangle is non-right angled.

Case (ii). If $P = 60^\circ$ and $Q = 150^\circ$:-

then $P + Q + R > 180^\circ$, which is not possible.

Case (iii). If $P = 120^\circ$ and $Q = 150^\circ$:-

then $P + Q + R > 180^\circ$, which is not possible.

Case (iv). If $P = 120^\circ$ and $Q = 30^\circ$:→

$\therefore \quad R = 30^\circ$

$\therefore$ From equation (1):→

$r = \sin R \times 2R$

$= \frac{1}{2} \times 2$

$\Rightarrow \quad r = 1$ —— (ii)

$\therefore$ $\boxed{\text{Length of median} = \frac{1}{2}\sqrt{2p^2 + 2q^2 - r^2}}$

$= \frac{1}{2}\sqrt{2 \times 3 + 2 \times 1 - 1}$

$\therefore$ $\boxed{RS = \frac{\sqrt{7}}{2}}$ option (C)

Now,

area of $\Delta PQR = \frac{1}{2} \times PQ \times PR \times \sin P = \frac{1}{2} \times PE \times QR$

$\Rightarrow \quad 1 \times 1 \times \frac{\sqrt{3}}{2} = PE \times \sqrt{3}$

$\Rightarrow \quad PE = \frac{1}{2}$ —— (iii)

$\therefore \quad OE = \frac{1}{3} PE$

$\Rightarrow \quad \boxed{OE = \frac{1}{6}}$ Ans. (A)

$\therefore$ Radius of incircle $= \dfrac{\Delta}{s}$

$$= \frac{2\Delta}{p+q+r}$$

$$= \frac{2 \times \frac{\sqrt{3}}{4}}{\sqrt{3}+1+1}$$

$$= \frac{\sqrt{3}}{2(2+\sqrt{3})} \times \frac{2-\sqrt{3}}{2-\sqrt{3}}$$

$$\boxed{r = \frac{\sqrt{3}}{2}(2-\sqrt{3})}$$ Ans (B)

Area of $\Delta OSE = \frac{1}{2} SE \times OE \times \sin \angle SEO$

$$= \frac{1}{2} \times \frac{1}{2} \times \frac{1}{6} \times \sin 60^\circ$$

$$\boxed{\text{Area of } \Delta OSE = \frac{\sqrt{3}}{48} \text{ unit}^2}$$ Ans (D)