PYQ3

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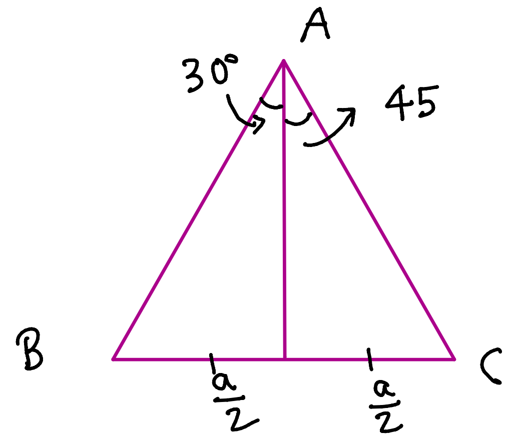
Question. In a ΔABC, the median to the side BC is of length $\frac{1}{\sqrt{11-6\sqrt{3}}}$ and it divides the angle A into angles of 30° and 45°. Find length of side BC. [Subjective Type, IIT-JEE 1985, 5]



Solution.

From Sine law in ΔADB,

$$\frac{a/2}{\sin 30^\circ} = \frac{AD}{\sin B} \quad \text{---} (1)$$

Also, from Sine law in ΔACD,

$$\frac{a/2}{\sin 45^\circ} = \frac{AD}{\sin C} \quad \text{---} (2)$$

Now, dividing equation (1) by equation 2:→

$$\frac{\sin 45^\circ}{\sin 30^\circ} = \frac{\sin C}{\sin B}$$

$$\Rightarrow \quad \frac{1/\sqrt{2}}{1/2} = \frac{c}{b}$$

$\left[\text{Using Sine law } \frac{\sin C}{\sin B} = \frac{c}{b}\right]$

$$\Rightarrow \quad c = b\sqrt{2} \quad \text{---} (3)$$

Now, in ΔABC,

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

$$\Rightarrow \quad a^2 = b^2 + (b\sqrt{2})^2 - 2b \times b\sqrt{2} \times \cos 75^\circ$$

$$\Rightarrow \quad a^2 = 3b^2 - 2\sqrt{2}\, b^2 \times \left(\frac{\sqrt{3}-1}{2\sqrt{2}}\right)$$

$$\Rightarrow \quad a^2 = b^2(4-\sqrt{3}) \quad \text{--- } (4)$$

Now, in ΔADB, using Sine Law: $\rightarrow$

$$\frac{a/2}{\sin 30^\circ} = \frac{AD}{\sin B} \quad \text{--- } (5)$$

By Sine Law, $\dfrac{b}{\sin B} = \dfrac{a}{\sin A} = 2R$ --- (6)

$\therefore$ From equation (5): $\rightarrow$

$$a = \frac{AD}{b/2R}$$

$$\Rightarrow \quad b = AD \times \frac{2R}{a} = AD \times \frac{1}{\sin A}$$

$$= \frac{AD}{\sin 75^\circ}$$

$$\Rightarrow \quad b^2 = \frac{AD^2}{\sin^2 75^\circ} = \frac{1}{11-6\sqrt{3}} \times \left(\frac{2\sqrt{2}}{\sqrt{3}+1}\right)^2$$

$$= \frac{8}{(4+2\sqrt{3})(11-6\sqrt{3})} \quad \text{--- } (7)$$

$\therefore$ From equation 4,

$$a^2 = (4-\sqrt{3}) \times \frac{8}{(4+2\sqrt{3})(11-6\sqrt{3})}$$

$$\Rightarrow \quad \boxed{a = BC = 2}$$

Ans.